# GPDs at EIC: Fits, Modelling, and opportunities

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March $18^{th}$, 2021

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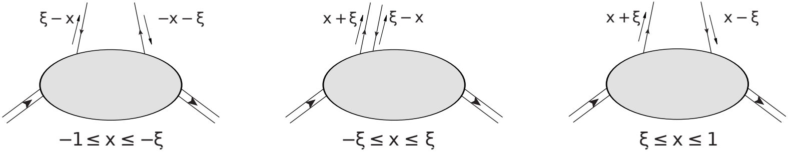
  - "hadron-parton" amplitudes which depend on three variables $(x, \xi, t)$ and a scale $\mu$,




  - ★ $x$: average momentum fraction carried by the active parton
  - ★ $\xi$: skewness parameter $\xi \simeq \frac{x_B}{2-x_B}$
  - ★ $t$: the Mandelstam variable

- Generalized Parton Distributions (GPDs):
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  - are defined in terms of a non-local matrix element,

$$\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$
$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\bar{u}\gamma^+u + E^q(x,\xi,t)\bar{u}\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u\right].$$

$$\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\gamma_5\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$
$$= \frac{1}{2P^+}\left[\tilde{H}^q(x,\xi,t)\bar{u}\gamma^+\gamma_5 u + \tilde{E}^q(x,\xi,t)\bar{u}\frac{\gamma_5\Delta^+}{2M}u\right].$$

D. Müller *et al.*, Fortsch. Phy. 42 101 (1994)
X. Ji, Phys. Rev. Lett. **78**, 610 (1997)
A. Radyushkin, Phys. Lett. **B380**, 417 (1996)

4 GPDs without helicity transfer + 4 helicity flip GPDs

- Generalized Parton Distributions (GPDs):
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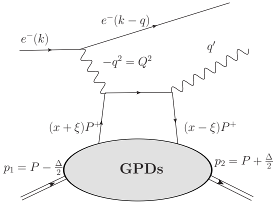
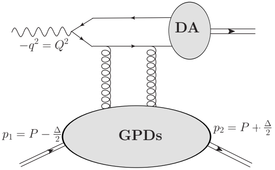
  - are related to PDF in the forward limit $H(x, \xi = 0, t = 0; \mu) = q(x; \mu)$
  - are universal, *i.e.* are related to the Compton Form Factors (CFFs) of various exclusive processes through convolutions

$$\mathcal{H}(\xi, t) = \int \mathrm{d}x \; C(x, \xi) H(x, \xi, t)$$

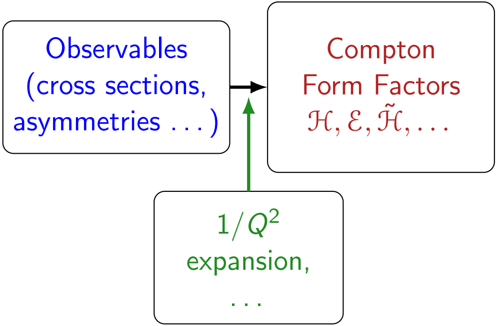


# Experimental connection to GPDs

Observables
(cross sections,
asymmetries . . . )

# Experimental connection to GPDs



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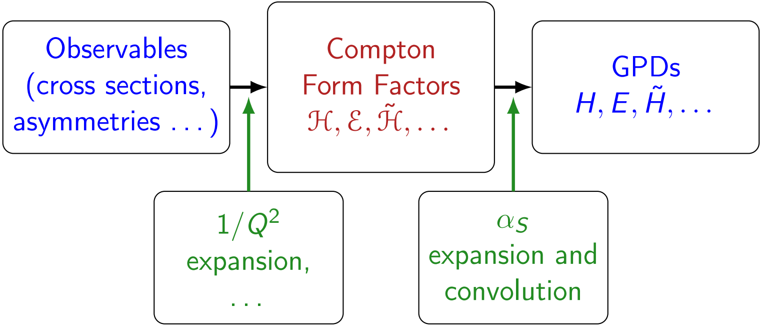
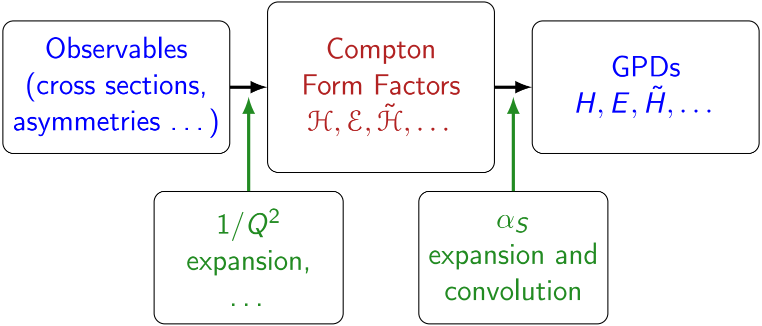
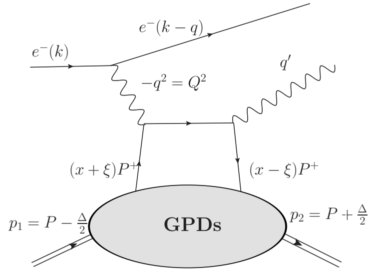




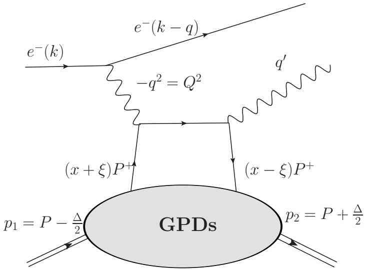

- CFFs play today a central role in our understanding of GPDs
- Extraction generally focused on CFFs

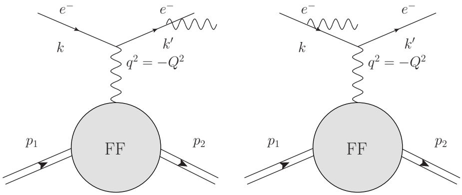
# Deep Virtual Compton Scattering




- Best studied experimental process connected to GPDs

  $\rightarrow$ Data taken at Hermes, Compass, JLab 6, JLab 12



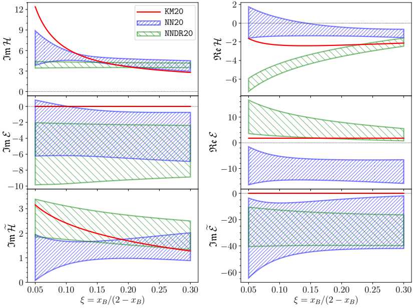




- Best studied experimental process connected to GPDs

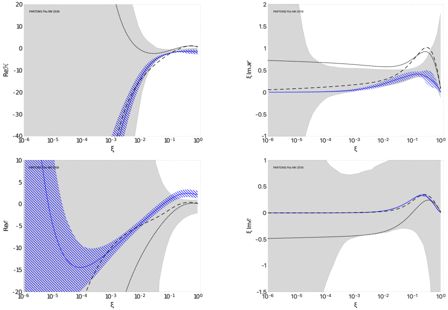
  → Data taken at Hermes, Compass, JLab 6, JLab 12
- Interferes with the Bethe-Heitler (BH) process
  - Blessing: Interference term boosted w.r.t. pure DVCS one
  - Curse: access to the angular modulation of the pure DVCS part difficult

M. Defurne *et al.*, Nature Commun. 8 (2017) 1, 1408

# Recent CFF extractions




M. Cuìc *et al.*, PRL 125, (2020), 232005




H. Moutarde *et al.*, EPJC 79, (2019), 614

- Recent effort on bias reduction in CFF extraction (ANN)

  additional ongoing studies, J. Grigsby *et al.*, arXiv:2012.04801

- Studies of ANN architecture to fulfil GPDs properties (dispersion relation,polynomiality,. . . )
- Recent efforts on propagation of uncertainties (allowing impact studies for EIC and EICC)

- At all order in $\alpha_S$, dispersion relations relate the real and imaginary parts of the CFF.

I. Anikin and O. Teryaev, PRD 76 056007
M. Diehl and D. Ivanov, EPJC 52 (2007) 919-932

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- For instance at LO:

$$Re(\mathcal{H}(\xi,t)) = \frac{1}{\pi}\int_{-1}^{1} \mathrm{d}x\ Im(\mathcal{H}(x,t))\left[\frac{1}{\xi - x} - \frac{1}{\xi + x}\right] + \underbrace{2\int_{-1}^{1} \mathrm{d}\alpha \frac{D(\alpha,t)}{1-\alpha}}_{\text{Independent of } \xi}$$

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- For instance at LO:

$$\underbrace{Re(\mathcal{H}(\xi,t))}_{\text{Extracted from data}} = \frac{1}{\pi}\int_{-1}^{1}\mathrm{d}x \underbrace{Im(\mathcal{H}(x,t))}_{\text{Extracted from data}} \left[\frac{1}{\xi-x}-\frac{1}{\xi+x}\right]+2\int_{-1}^{1}\mathrm{d}\alpha\frac{D(\alpha,t)}{1-\alpha}$$

- $D(\alpha,t)$ is related to the EMT (pressure and shear forces)

M.V. Polyakov PLB 555, 57-62 (2003)

# Dispersion relation and the D-term

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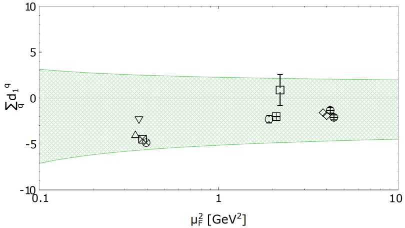


figure from H. Dutrieux *et al.*, accepted in EPJC, arXiv 2101.03855

- Subtraction constant obtained through ANN fit
- World data yield a result compatible with 0

# From CFF to GPDs

- Polynomiality Property:

$$\int_{-1}^{1} \mathrm{d}x\, x^m H^q(x,\xi,t;\mu) = \sum_{j=0}^{\left[\frac{m}{2}\right]} \xi^{2j} C_{2j}^q(t;\mu) + mod(m,2)\xi^{m+1} C_{m+1}^q(t;\mu)$$

X. Ji, J.Phys.G 24 (1998) 1181-1205
A. Radyushkin, Phys.Lett.B 449 (1999) 81-88

Special case :

$$\int_{-1}^{1} \mathrm{d}x\ H^q(x,\xi,t;\mu) = F_1^q(t)$$

Lorentz Covariance

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

$$\left| H^q(x,\xi,t) - \frac{\xi^2}{1-\xi^2} E^q(x,\xi,t) \right| \leq \sqrt{\frac{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}{1-\xi^2}}$$

A. Radysuhkin, Phys. Rev. **D59**, 014030 (1999)
B. Pire *et al.*, Eur. Phys. J. **C8**, 103 (1999)
M. Diehl *et al.*, Nucl. Phys. **B596**, 33 (2001)
P.V. Pobilitsa, Phys. Rev. **D65**, 114015 (2002)

Positivity of Hilbert space norm

# GPD Properties

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

Positivity of Hilbert space norm

- Support property:

$$x \in [-1; 1]$$

M. Diehl and T. Gousset, Phys. Lett. **B428**, 359 (1998)

Relativistic quantum mechanics

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- Soft pion theorem (pion GPDs only)

M.V. Polyakov, Nucl. Phys. **B555**, 231 (1999)
CM *et al.*, Phys. Lett. **B741**, 190 (2015)

Axial-Vector WTI

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  Relativistic quantum mechanics

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- Scale evolution property:
  $\rightarrow$ generalization of DGLAP and ERBL evolution equations

  D. Müller *et al.*, Fortschr. Phys. 42, 101 (1994)

  Renormalization

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- Scale evolution property:

Renormalization

**Problem**

- There is no model (until now) fulfilling *a priori* all these constraints.
- Lattice QCD computations remain very challenging.

# DVCS Deconvolution Problem I

$$\mathcal{H}(\xi, t, Q^2) = \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} C\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu)\right) H(x, \xi, t, \mu)$$

**Definition of the problem**

Being given $\mathcal{H}$ on a large kinematical range and with excellent precision, is it possible to recover $H$ unambiguously?

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- Not a new problem (already raised in the 1990s), but it remains open, generally speaking.
- At LO, the answer is definitely no. We can find 0-order shadow GPDs $h_0$ providing that
  - $h_0(x, 0, 0) = 0$ and $h_0(x, x, t) = 0$
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Role of higher orders corrections and evolution equations?

# DVCS Deconvolution Problem II

Hervé Dutrieux

- Singularity structure of DVCS coefficient function
  $\rightarrow$ proof that the convolution is not invertible
- there exist NLO shadow GPD $h_1$
- Evolution: it yields contributions of $\sim \alpha_S^2$

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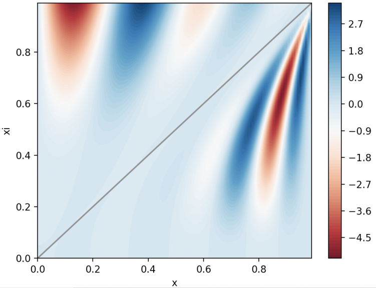
- Explicit construction of an example
- Vector space of solution $\rightarrow$ many shapes allowed



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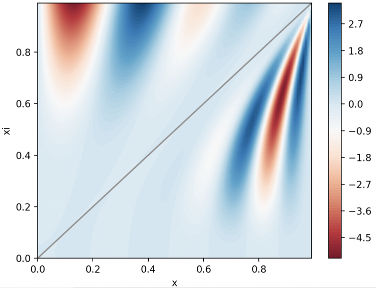
**A way out**

Shadow GPD are process dependent
Multichannel analyses offer a way out



# PARTONS and Gepard

## PARTONS

partons.cea.fr

B. Berthou *et al.*, EPJC 78 (2018) 478

## Gepard

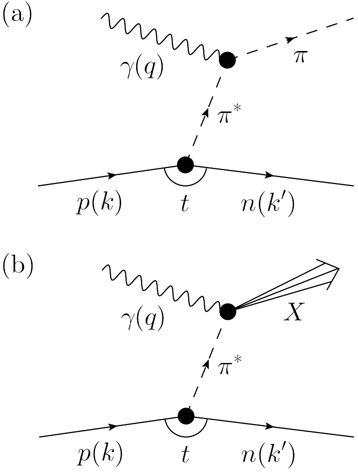
calculon.phy.hr/gpd/server/index.html

K. Kumericki, EPJ Web Conf. 112 (2016) 01012

- Similarities : NLO computations, BM formalism, ANN, ...
- Differences : models, evolution, dissemination, ...

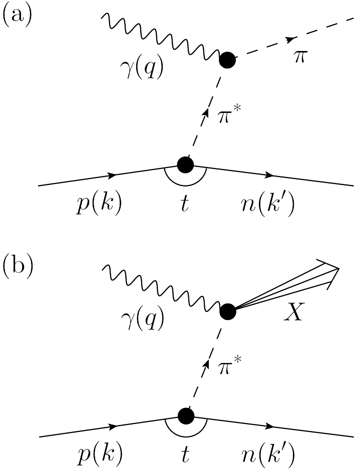
### Physics impact

These integrated softwares are the mandatory path toward reliable multichannel analyses.



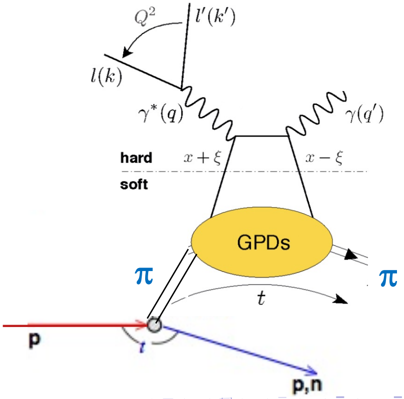

- virtual pion targets envisioned for FF and PDF measurement at an EIC

# The Sullivan process: targeting the pion




- virtual pion targets envisioned for FF and PDF measurement at an EIC
- would DVCS be possible ?

see C. Keppel talk at CFNS workshop 06/2020



# PARTONS: a way to assess feasibility

José Manuel Morgado Chavez

- Elaborating on a previous paper by D. Amrath, M. Diehl and J.-P. Lansberg

  D. Amrath *et al.*, Eur.Phys.J.C 58 (2008) 179-192
- State-of-the-art pion GPD model
  - Positivity and Polynomiality obtained by construction
  - $t$ dependence fixed to latest available experimental data for EMFF and GFF

    G.M. Huber *et al.*, Phys.Rev.C 78 (2008) 045203
    S. Kumano *et al.*, Phys.Rev.D 97 (2018) 1, 014020
  - Forward limit built from state-of-the-art DSE computations

    M. Ding *et al.*, Phys.Rev.D 101 (2020) 5, 054014
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Work in progress, stay tuned for our results

# Summary & Conclusion

## Summary

- Global fits of CFFs have been pursued through ANN
- The leap from CFFs to GPDs is complicated because of both theoretical constraints and non-invertibility of the convolution with the coefficient function
- Way out : multichannel analysis (DVMP, DDVCS, Multi-particle production. . . )

## Conclusion

- Multichannel analyses is the way to go for global GPD fits
- Other opportunities at an EIC beside the nucleon (pion, nuclei)
- Work in progress on the Sullivan process

Thank you for your attention